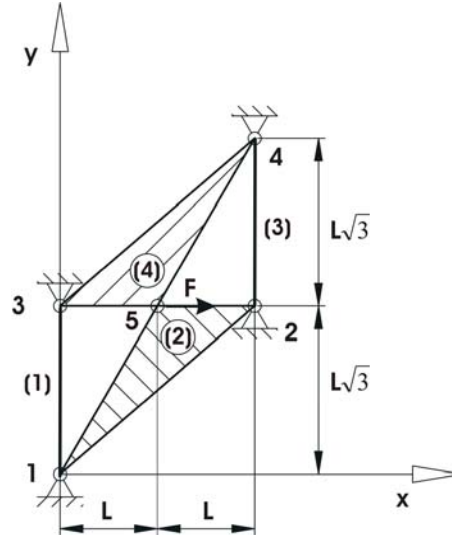


Zadatak 1

Za model planarne mehaničke strukture prikazane na slici potrebno je primenom metoda konačnih elemenata:

1. Sastaviti ukupnu matricu masa.
2. Odrediti sopstvene frekvence sistema i nacrtati osnovne oblike oscilovanja.



Matrica krutosti sistema je data sa:

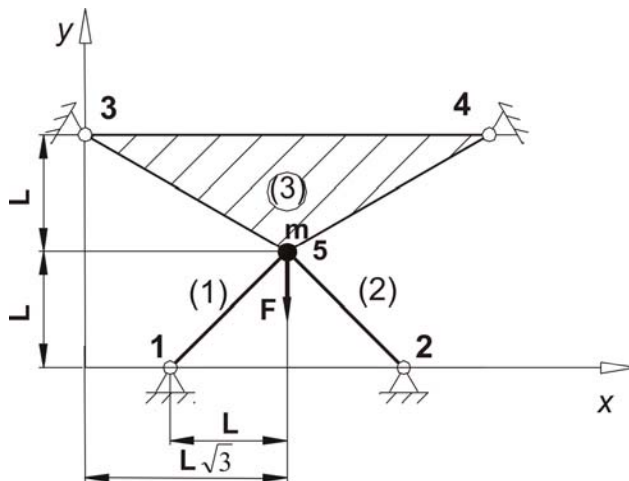
$$K = \frac{EA}{L} \begin{bmatrix} \frac{\sqrt{3}}{16} & 0 & \frac{\sqrt{3}}{16} & \frac{-3}{16} & 0 & 0 & 0 & 0 & \frac{-\sqrt{3}}{8} & \frac{3}{16} \\ 0 & \frac{25\sqrt{3}}{48} & \frac{-3}{16} & \frac{3\sqrt{3}}{16} & 0 & \frac{-\sqrt{3}}{3} & 0 & 0 & \frac{3}{16} & \frac{-3\sqrt{3}}{8} \\ \frac{\sqrt{3}}{16} & \frac{-3}{16} & \frac{5\sqrt{3}}{8} & \frac{-3}{8} & 0 & 0 & 0 & 0 & \frac{-11\sqrt{3}}{16} & \frac{9}{16} \\ \frac{-3}{16} & \frac{3\sqrt{3}}{16} & \frac{-3}{8} & \frac{17\sqrt{3}}{24} & 0 & 0 & 0 & \frac{-\sqrt{3}}{3} & \frac{9}{16} & \frac{-9\sqrt{3}}{16} \\ 0 & 0 & 0 & 0 & \frac{5\sqrt{3}}{8} & \frac{-3}{8} & \frac{\sqrt{3}}{16} & \frac{-3}{16} & \frac{-11\sqrt{3}}{16} & \frac{9}{16} \\ 0 & \frac{-\sqrt{3}}{3} & 0 & 0 & \frac{-3}{8} & \frac{17\sqrt{3}}{24} & \frac{-3}{16} & \frac{3\sqrt{3}}{16} & \frac{9}{16} & \frac{-9\sqrt{3}}{16} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{16} & \frac{-3}{16} & \frac{\sqrt{3}}{16} & 0 & \frac{-\sqrt{3}}{8} & \frac{3}{16} \\ 0 & 0 & 0 & \frac{-\sqrt{3}}{3} & \frac{-3}{16} & \frac{\sqrt{3}}{16} & 0 & \frac{25\sqrt{3}}{48} & \frac{3}{16} & \frac{-3\sqrt{3}}{8} \\ \frac{-\sqrt{3}}{8} & \frac{3}{16} & \frac{-11\sqrt{3}}{16} & \frac{9}{16} & \frac{-11\sqrt{3}}{16} & \frac{9}{16} & \frac{-\sqrt{3}}{8} & \frac{3}{16} & \frac{13\sqrt{3}}{16} & \frac{-3}{8} \\ \frac{3}{16} & \frac{-3\sqrt{3}}{16} & \frac{9}{16} & \frac{-9\sqrt{3}}{16} & \frac{9}{16} & \frac{-9\sqrt{3}}{16} & \frac{3}{16} & \frac{-\sqrt{3}}{8} & \frac{-3}{16} & \frac{15\sqrt{3}}{8} \\ \frac{3}{16} & \frac{-3\sqrt{3}}{16} & \frac{9}{16} & \frac{-9\sqrt{3}}{16} & \frac{9}{16} & \frac{-9\sqrt{3}}{16} & \frac{3}{16} & \frac{-\sqrt{3}}{8} & \frac{-3}{16} & \frac{15\sqrt{3}}{8} \end{bmatrix}$$

Napomena: konačni elementi u obliku štapa imaju površinu poprečnog preseka A; debljina trougaonih konačnih elemenata je $t=A/L$; svi elementi su od materijala čiji je modul elastičnosti E, gustina ρ , a Puasonov koeficijent $\nu=1/3$.

Zadatak 2

Za model planarne mehaničke strukture prikazane na slici, potrebno je primenom metode konačnih elemenata:

1. Sastaviti ukupnu matricu masa.
2. Odrediti sopstvene frekvence sistema



Matrica krutosti sistema je data sa:

$$K = \frac{EA}{L} \begin{bmatrix} \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} \\ \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} \\ 0 & 0 & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} \\ 0 & 0 & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{3\sqrt{3}}{16} & -\frac{3}{16} & \frac{3\sqrt{3}}{16} & 0 & -\frac{3\sqrt{3}}{16} & \frac{3}{16} \\ 0 & 0 & 0 & 0 & -\frac{3}{16} & \frac{5\sqrt{3}}{16} & 0 & \frac{\sqrt{3}}{4} & \frac{3}{16} & -\frac{9\sqrt{3}}{16} \\ 0 & 0 & 0 & 0 & \frac{3\sqrt{3}}{16} & 0 & \frac{3\sqrt{3}}{16} & \frac{3}{16} & -\frac{3\sqrt{3}}{16} & -\frac{3}{16} \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{4} & \frac{3}{16} & \frac{5\sqrt{3}}{16} & -\frac{3}{16} & -\frac{9\sqrt{3}}{16} \\ -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & -\frac{3\sqrt{3}}{16} & \frac{3}{16} & -\frac{3\sqrt{3}}{16} & -\frac{3}{16} & \frac{\sqrt{2}}{2} + \frac{3\sqrt{3}}{8} & 0 \\ -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & \frac{3}{16} & -\frac{9\sqrt{3}}{16} & -\frac{3}{16} & -\frac{9\sqrt{3}}{16} & 0 & \frac{\sqrt{2}}{2} + \frac{9\sqrt{3}}{8} \end{bmatrix}$$

Napomena: konačni elementi u obliku štapa imaju površinu poprečnog preseka A; debljina trougaonih konačnih elemenata je $t=A/L$; svi elementi su od materijala čiji je modul elastičnosti E, gustina ρ , a Puasonov koeficijent $\nu=1/3$.